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Metal–insulator transition in 2D as a quantum phase transition

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Abstract
We discuss the metal–insulator transition phenomenon in two dimensions in terms of a quantum critical point that controls a range of the low temperature insulator region as well as the usual quantum critical sector. We show that this extended range of criticality permits a determination of both the dynamical critical exponent z and the correlation length critical exponent ν from published data from a single experiment in the insulator critical region. The resulting value of the product $z\nu$ is consistent with the temperature dependence of the resistance in the quantum critical sector. This provides strong quantitative evidence for the presence of a quantum critical point.

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(Some figures in this article are in colour only in the electronic version)

We focus in this paper on a phenomenological description of the metal–insulator (MI) transition phenomena in two dimensions (2D) in the case of disorder due to potential scattering. The transition occurs as the density is tuned through a critical value $n = n_c$. Analysis of experimental results for the T dependence of the resistivity of Si, GaAs and other 2D semiconductor structures in the vicinity of the observed separatrix is suggestive of a quantum phase transition [1–3].

Let us recall the main features of a quantum phase transition. This is a phase transition in the ground state at $T = 0$ as a tuning parameter Δ is varied through a critical point. By definition the tuning parameter Δ is zero at the quantum critical point (QCP). In the metal–insulator transition case, Δ is related to the carrier density n and the sign of Δ distinguishes between a metallic ground state ($\Delta > 0$) and an insulating ground state ($\Delta < 0$). We can consider $\Delta \sim n - n_c$.

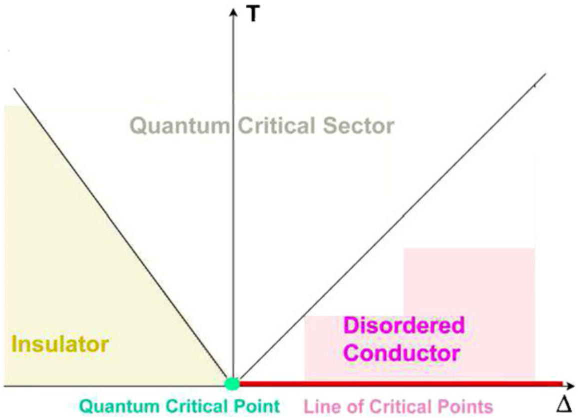

Figure 1. Phase diagram with a QCP at $\Delta = 0$ and $T = 0$ separating insulating and metallic phases of the ground state. For a finite range of non-zero temperatures and Δ sufficiently close to the QCP, critical exponents of both the quantum critical sector and the insulating critical sector are determined by the QCP.

Of course experiments are done at a finite temperature. To test the hypothesis of a QCP we require a theory which is valid at low temperature and which contains the signature of a QCP. It must be guaranteed from the outset that the theory includes $T = 0$ in its domain of validity. Therefore, in addition to Δ, the temperature must also be a scaling variable. These are the leading relevant variables in a scaling description of physical properties in the vicinity of the QCP [2, 3].

We will be giving a phenomenological scaling description of the vicinity of the QCP. It is implicit that there exists a renormalization group (RG) foundation from which scaling behavior of Δ and T could be determined. However, in the vicinity of the QCP where both Δ and T are small the solutions of the RG equations for these variables are generic. The scaling solution for Δ leads to a density correlation length ξ and the solution for T leads to the thermal length L_T. The density correlation length $\xi \sim |\Delta|^{-\nu}$ characterizes correlations in the vicinity of the transition at $T = 0$. At finite T the thermal length L_T varies as $1/T^{1/z}$ [2, 3]. At $\Delta = 0$ or for any $\xi \gg L_T$, the L_T determines the rate of decay of correlations. The vicinity of the QCP will show critical behavior and physical properties will be determined by the ratio Δ/T, or equivalently by the ratio L_T/ξ.

There are three critical sectors associated with the QCP (see figure 1). The first sector is the low-T disordered conductor where $\Delta > 0$ and Δ/T is large. This low-T disordered conductor is the region where perturbative RG can be applied and this approach has yielded the very important result that electron–electron interactions stabilize the metallic phase [4–7]. However this region is extremely complex due to long range quantum-induced correlations that cause nonanalytical response functions [8–11] and consequently an additional source of criticality. In fact the entire line with $T = 0$ to the right of the QCP is critical. This raises complications when attempting to determine critical properties by approaching the QCP from the metallic critical sector. However the two other critical sectors are not affected by these nonanalyticities and thus give more direct information on the QCP. For this reason we will not consider further the metallic sector in this paper.

The second critical sector has Δ/T small with Δ either positive or negative [2, 3]. We will refer to this sector as the quantum critical sector (QCS)[5]. The third sector, with Δ/T large

[5] In the literature this sector is also referred to as the quantum critical region.

and $\Delta < 0$, is the low-T insulating critical sector (ICS). Criticality in the QCS is expected but we point out that new information on criticality is also obtainable from the ICS.

The existence of a single QCP controlling both the QCS and ICS implies that the resistivity in both sectors near the QCP is given by a single universal function of Δ/T. This universal function can have different limiting forms for large and small Δ/T, but critical exponents in the two sectors must be the same since they are intrinsic properties of the QCP.

Thus far we have considered only the most relevant variables near the QCP, Δ and T. Another quantity relevant for the metal–insulator phenomena is the strength of the electron–electron interaction, which is accounted for by the electron–electron scattering amplitude γ_{ee}.

The scaling equation for the resistance in the vicinity of the QCP can be expressed in the form $\mathrm{d}\rho/\mathrm{d}\log L = f(\rho, \gamma_{ee})$, with L being the length of the system or the thermal length L_T. Experimental evidence shows that γ_{ee} varies slowly over the temperature range in the vicinity of the bifurcation [12]. We assume that the effect of this weak variation of γ_{ee} is to tilt the separatrix. The effect of a tilted separatrix can be accounted for empirically and subtracted from the experimental data near the bifurcation and in the insulating sector. Scaling equations of the simpler form $\mathrm{d}\rho/\mathrm{d}\log L = f(\rho)$ can then be used [13].

In order to determine an explicit scaling equation we take $\mathrm{d}\rho/\mathrm{d}\log L$ in the insulator to be represented as a series in powers of ρ and $\log\rho$. The series must be positive and monotonically increasing with ρ. Also ρ must remain finite for all finite L, diverging only in the limit $L \to \infty$. The leading term in the series at large L consistent with these conditions is $k\rho\log\rho$ with k being a positive constant. Otherwise ρ would diverge at finite L [14, 15]. The scaling equation at large ρ is then

$$\frac{\mathrm{d}\log\rho}{\mathrm{d}\log L} = k\log\rho\left(1 + \mathcal{O}\left(\frac{1}{\log\rho}\right)\right). \tag{1}$$

The solution of equation (1) at large L is $\log\rho \sim L^k$. The dominant variation of ρ with L is expected to be exponential in the insulating limit, $\rho \propto \exp(L/\xi)$, with ξ being the correlation length [16]. Taking the leading variation of $\log\rho$ to be linear in L then determines $k = 1$. Equivalently ρ has a log-normal probability distribution in the strong disorder limit.

To describe the T dependence of the resistance we replace L by the thermal length $L_T \propto 1/T^{1/z}$, where z is the dynamical critical exponent of the QCP. Including the corrections in inverse powers of $\log\rho$ in equation (1) we obtain the solution

$$\rho(T) = \rho_0[1 + \mathcal{O}((T/T_0)^{1/z})]\exp[(T_0/T)^{1/z}]. \tag{2}$$

The T dependence of equation (2) is dominated by the exponential at low T so that the relatively weak T dependence of the prefactor can be neglected. The resistance in the ICS is thus described by

$$\rho(T) = \rho_0(n)\exp[(T_0(n)/T)^{1/z}] \tag{3}$$

and fits to the experimental data in the ICS will then give the dynamical critical exponent z.

In fact we can also obtain the critical exponent ν of the correlation length from the same data. The exponential in equation (3) is also $\exp[-(L_T/\xi)]$ [16] which implies

$$(T_0(n)/T)^{1/z} = L_T/\xi. \tag{4}$$

Then the density dependence of the temperature scale $T_0(n)$ is

$$T_0(n) \propto |(n - n_c)/n_c|^{z\nu} = |\delta_n|^{z\nu}. \tag{5}$$

Thus analysis of the T dependence of $\rho(T)$ followed by analysis of the n dependence of $T_0(n)$ will yield z and ν, respectively.

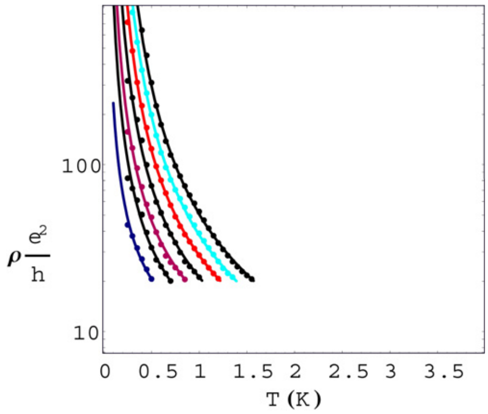

Figure 2. Data points taken from [20] in the ICS for densities from 0.7156 (top) to 0.8464 $\times 10^{11}$ cm^{-2}. The high resistance cutoff is taken at $\rho_{\text{cut}} = 20$. Solid lines are best fits using equation (3) with z and the density dependent $T_0(n)$ as the fitting parameters.

Experimental results for the resistivity in the low-T insulating regime have been reported for a variety of 2D systems [17–23]. To compare theory and experiment we chose to analyze data for a Si sample from [20]. This reference contains T-dependent resistivity data over a wide range covering the insulating sector and the region of the bifurcation.

We account for a tilted separatrix in the data by assuming it to have the functional form $\log(\rho_{\text{sep}}(T)/\rho_c) = m_{\text{sep}}T$. By interpolating between adjacent insulating and metallic curves and extrapolating to low T we deduce $m_{\text{sep}} = -0.1608$ K^{-1} with critical resistivity $\rho_c = 3.757(h/e^2)$ and critical density $n_c = 0.9481 \times 10^{11}$ cm^{-2}. In fitting the scaling equations to the experimental data, we first divide all the $\rho(T)$ data points by $\rho_{\text{sep}}(T)$ to remove the effect of the separatrix tilt [14].

For the insulating range of ρ we fitted the resulting data points to equation (3). To ensure that fitted data lie well inside the insulating region, we selected low T, high $\rho(T)$ points by restricting $\rho(T) > \rho_{\text{cut}} = 20$. Numerical least squares methods determined a value of z for each of the curves in this region. To test consistency with respect to the range of fit the procedure was repeated for different values of ρ_{cut} up to $\rho_{\text{cut}} = 30$. Analysis of the resulting set of values of z gave a mean value and standard deviation $z = 2.05 \pm 0.10$. The curves in the ICS for $\rho(T) > 20$ in figure 2 show equation (3) for this value of z. All the data points shown satisfy $L_T/\xi > 3$.

We next determine the critical exponent ν from the same data. Fixing $z = 2.05 \pm 0.10$, we made least squares fits of the insulating data to equation (5) to obtain $T_0(n)$ for each density curve. The fits to the data are given in figure 3 where the $T_0(n)$ points are shown as triangles. From equation (5) it follows that $T_0(n)$ varies as $|\delta_n|^{z\nu}$. Figure 3 confirms this power law dependence. The solid line is the best fit of $[d(-\delta_n)]^{z\nu}$ to the triangles, with $z = 2.05$ fixed. The best fitting parameters are $d = 14.9$ and $\nu = 1.17$. Since the uncertainty in z is ± 0.10 a corresponding error estimate for ν is obtained by repeating this fitting with $z = 1.95$ and 2.15, yielding $\nu = 1.20$ and 1.15, respectively. The value of the product $z\nu$ is then 2.4 ± 0.1.

The determination of a scaling equation for the resistivity in the QCS is straightforward. After correcting for the tilted separatrix, scale invariance of the resistivity at a critical density implies $\mathrm{d}\rho/\log L = 0$ at a critical resistivity ρ_c. Taking $\mathrm{d}\rho/\log L$ to vanish linearly in $\rho - \rho_c$ with a slope $1/\nu$ then provides an explicit solution near the bifurcation. Applying this

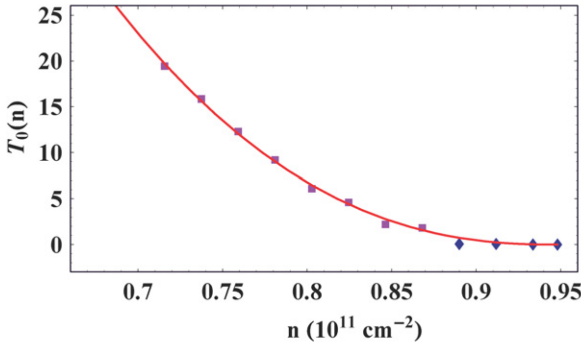

Figure 3. Fits of experimental data within the ICS from [20] to $T_0(n)$ of equation (5) (squares). Solid line is a fit to the squares by the function $d(-\delta_n)^{z\nu}$ with ν being a free parameter. The best fit value of ν is $\nu = 1.17 \pm 0.3$. Diamonds show $\tilde{T_0}(n)$ resulting from fits of equation (9) to the experimental data within the QCS from [20].

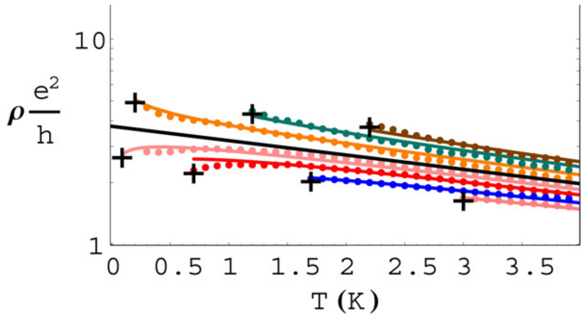

Figure 4. Experimental data points in QCS for densities from 0.8900 (top) to 1.0426×10^{11} cm^{-2}. Solid lines show equation (8) using the value $z\nu = 2.4$ as determined from the ICS with $\rho_c = 3.757$ determined from the separatrix (solid straight line). The theoretical QCS curves terminate for $L_T/\xi = [c|\delta_n|/T^{1/z\nu}]^\nu = 1/3$.

argument to the vanishing of $\log(\rho/\rho_c)$ rather than to ρ/ρ_c [24], and integrating the scaling equation from a starting L_s to a final L results in

$$\log(\rho/\rho_c) = \log(\rho_s/\rho_c)(L/L_s)^{1/\nu}. \quad (6)$$

Replacing L by L_T gives

$$\rho(T) = \rho_c \exp[\log(\rho(T_s)/\rho_c)(T_s/T)^{1/z\nu}] \quad (7)$$

$$= \rho_c \exp[(\tilde{T}_0(n)/T)^{1/z\nu}] \quad (8)$$

where we have identified

$$\tilde{T}_0(n) = (\rho(T_s)/\rho_c)^{z\nu} T_s. \quad (9)$$

It is a property of equation (8) that $\tilde{T}_0(n)$ is in fact independent of the starting T_s.

Equation (8) provides an important consistency check on our proposal that the bifurcation is due to a QCP and that the QCS and ICS share the same critical exponents. The procedure is as follows. We use equation 7 with $z\nu = 2.40 \pm 0.10$ as determined from the ICS and with ρ_c determined from the separatrix, to construct a theoretical $\rho(T)$ for the QCS. We compare this constructed $\rho(T)$ with the experimental data points from [20] lying within the region $L_T/\xi = [\rho(T_s)/\rho_c]^{\nu}(T_s/T)^{1/z} < 1/3$. Even though there are no fitting parameters for the T dependence, we see in figure 4 there is good agreement. Furthermore using the same $z\nu$ determines the temperature scale $\tilde{T}_0(n)$ in equation (9). This is plotted in figure 3 and is seen to fall on the same curve as the data from the ICS. This verifies the values of z and ν obtained from the ICS are consistent with the product $z\nu$ for the QCS and that the temperature scales $T_0(n)$ and $\tilde{T}_0(n)$ are the same.

In summary we have shown that the region of small Δ and small T is critical and is described by a scaling picture with two length scales, the thermal length L_T and the correlation length ξ. The QCP controls both the QCS at small L_T/ξ and the ICS at large L_T/ξ, and these two regions share the same critical exponents. Critical behavior in the vicinity of the bifurcation is expected, but the surprising point is that part of the insulating sector also contains information on criticality. As a consequence both exponents can be obtained from data in the insulating sector. A unified picture emerges for critical behavior in both the quantum critical sector and the insulating critical sector in the neighborhood of the quantum critical point.

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